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PATTERNS GENERATION AND TRANSITION MATRICES IN MULTI-DIMENSIONAL LATTICE MODELS

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Abstract. In this paper we develop a general approach for investigating pattern generation problems in multi-dimensional lattice models. Let $\mathcal{S}$ be a set of p symbols or colors, $\mathbf{Z}_N$ a fixed finite rectangular sublattice of $\mathbf{Z}^d$, $d \geq 1$ and N a d-tuple of positive integers. Functions $U : \mathbf{Z}^d \to \mathcal{S}$ and $U_N : \mathbf{Z}_N \to \mathcal{S}$ are called a global pattern and a local pattern on $\mathbf{Z}_N$, respectively. We introduce an ordering matrix $\mathbf{X}_N$ for Σ_N, the set of all local patterns on $\mathbf{Z}_N$. For a larger finite lattice $\mathbf{Z}_{\tilde{N}}$, $\tilde{N} \geq N$, we derive a recursion formula to obtain the ordering matrix $\mathbf{X}_{\tilde{N}}$ of $\Sigma_{\tilde{N}}$ from $\mathbf{X}_N$. For a given basic admissible local patterns set $\mathcal{B} \subset \Sigma_N$, the transition matrix $\mathbf{T}_N(\mathcal{B})$ is defined. For each $\tilde{N} \geq N$ denoted by $\Sigma_{\tilde{N}}(\mathcal{B})$ the set of all local patterns which can be generated from $\mathcal{B}$, the cardinal number of $\Sigma_{\tilde{N}}(\mathcal{B})$ is the sum of entries of the transition matrix $\mathbf{T}_{\tilde{N}}(\mathcal{B})$ which can be obtained from $\mathbf{T}_N(\mathcal{B})$ recursively. The spatial entropy $h(\mathcal{B})$ can be obtained by computing the maximum eigenvalues of a sequence of transition matrices $\mathbf{T}_n(\mathcal{B})$. The results can be applied to study the set of global stationary solutions in various Lattice Dynamical Systems and Cellular Neural Networks.

1. **Introduction.** Many systems have been studied as models for spatial pattern formation in biology, chemistry, engineering and physics. Lattices play important roles in modeling underlying spatial structures. Notable examples include models arising from biology [7, 8, 21, 22, 23, 33, 34, 35], chemical reaction and phase transitions [4, 5, 11, 12, 13, 14, 24, 41, 43], image processing and pattern recognition [11, 12, 15, 16, 17, 18, 19, 25, 40], as well as materials science[9, 20, 26]. Stationary patterns play a critical role in investigating of the long time behavior of related dynamical systems. In general, multiple stationary patterns may induce complicated phenomena of such systems.

In Lattice Dynamical Systems(LDS), especially Cellular Neural Networks (CNN), the set of global stationary solutions (global patterns) has received considerable attention in recent years (e.g.[1, 2, 6, 10, 27, 28, 29, 30, 31, 32, 36, 37]). When the

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